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Name of Examination : **FY Winter 2021** - (Preview)

Course Code & Course Name : **SH101U - Differential Calculus**

Generated At : **18-05-2022 11:29:13**

Maximum Marks : **60** Duration : **3 Hrs**

Edit Print View Answer Key Close **Answer Key Submission Type:** Marking scheme with model answers and solutions of numerical

Instructions:

1. *All questions are compulsory.*
2. *Illustrate your answer with suitable figures/sketches wherever necessary.*
3. *Assume suitable additional data; if required.*
4. *Use of logarithmic table, drawing instruments and non programmable calculators is allowed.*
5. *Figures to the right indicate full marks.*

1) Solve all Sub-questions.

a) If $y = \left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$, then show that [5]

$(1+x^2)\,y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0,$

b) [5]

Verify $JJ' = 1$, for $x = u\cos v, y = u\sin v$, where $J = \frac{\partial(x,y)}{\partial(u,v)}$.

c) [4]

Find the inverse of the matrix by Gauss-Jordan method

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 5 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

2) Solve any three sub-questions

a) Find the modulus and argument of $(1-i)^{1+i}$. Consider the principal value only. [4]

b) Discuss maxima and minima of the function. [4]

$$f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$$

c) If $\tan\alpha = \tan x \tanh y$ and $\tan\beta = \cot x \tanh y$, then prove that [4]

$$\tan(\alpha+\beta) = \sinh 2y \operatorname{cosec} 2x$$

d) Expand $f(x) = log\ cos\ (\frac{\pi}{4} + x)$ in ascending powers of x by Taylor's theorem and hence find the value of $log\ cos\ (48^0)$. [4]

3) Solve any three sub-questions

a) In estimating the value of $f(x,y) = x^2 + y^3 - 3xy$ at the point $x = 2, y = 1$ an error is made in the measuring scale with errors +0.1 and -0.1 in x & y respectively, then find the approximate error in the value of f. [4]

b) Find eigen values & one eigen vector corresponding to an integer eigen value of the matrix [4]

$$A = \begin{bmatrix} 3 & 0 & 1 \\ -3 & 4 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

c) Examine for consistency and if consistent then solve it [4]

$$2x_1 + x_2 - x_3 + 3x_4 = 8; \quad x_1 + x_2 + x_3 - x_4 = -2$$
$$3x_1 + 2x_2 - x_3 = 6; \quad 4x_2 + 3x_3 + 2x_4 + 8 = 0$$

d) Evaluate $\lim_{x \to -\infty} xe^x$. [4]

4) Solve any three sub-questions

a) Obtain the parametric and symmetric equations of the line L that passes through the point (1, -2, 4) and is parallel to $v = \langle 2, 4, -4 \rangle$. [4]

b) Show that the family of parabolas $y^2 = 4a(x + a)$ is self-orthogonal. [4]

c) If $\sin(\alpha + i\beta) = r(\cos x + i \sin x)$ then prove that [4]

(i) $r^2 = \frac{1}{2}(\cosh 2\beta - \cos 2\alpha)$ (ii) $\tan x = \tanh \beta \cot \alpha$

d) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. [4]

5) Solve any two sub-questions

a) (i) Find a Cartesian equation for the equation in Cylindrical coordinates $z = r^2$, and identify the surface. [5]

(ii) Express the point with cylindrical coordinates $(4, 2\pi/3, -2)$ in rectangular coordinates.

b) Verify Cayley-Hamilton theorem for [5]

$$\begin{bmatrix} 1 & 4 & 0 \\ -3 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

c) If $x = e^u \tan v$ and $y = e^u \sec v$, then show that [5]

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) = 0.$$

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